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New Wall-Reflection Model Applied to the Turbulent Impinging Jet

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Introduction

WITH the continuing rapid decrease in the cost of computer resources, computations of turbulent flows are progressively raising the level of physical models employed to represent turbulent momentum transport. Although, at present, most computations of aerodynamic flows still use models based on an effective turbulent viscosity, a growing minority adopt schemes that, instead, solve a set of rate equations for the turbulent stresses and, where appropriate, for the turbulent heat fluxes. Models of this type are known as second-moment (or second-order) closures. So far they have been applied mainly to free flows or to flows broadly parallel to walls and have established a track record of out performing eddy viscosity models, particularly where the streamlines are curved.

In second-moment closures for the turbulent stress field, a wall-reflection correction is conventionally added to the model of the pressure-strain correlation φ_{ij} in computing flow near walls. Its role is to reduce the level of turbulent velocity fluctuations normal to the wall and, through the strong intercoupling among the Reynolds stress components, to reduce generally the level of turbulent mixing. The various models of this process have been designed to produce approximately the correct relative levels of the Reynolds stresses in the near-wall, local-equilibrium region of the turbulent boundary layer or some other similar shear flow directed *parallel* to the wall (see Shih and Lumley[1] and Gibson and Launder[2]). When, however, the scheme of Ref. 2 was applied to the axisymmetric impinging jet[3,4] (see the broken lines in Figs. 1 and 2), it led to excessive levels of the turbulent stresses in the vicinity of the stagnation point. This anomalous behavior of the wall correction near stagnation points has also been recently noted by Murakami et al.[5] in a study of a three-dimensional buoyant jet in an enclosure and by Lea[6] in an in-cylinder flow. The present contribution proposes an alternative formulation of the part of the model giving rise to the aforementioned aberrant behavior.

Analysis

The pressure-strain correlation $\overline{p(\partial u_i/\partial x_j + \partial u_j/\partial x_i)}/\rho \equiv \varphi_{ij}$ that, as its name suggests, is the time-averaged product of the turbulent kinematic pressure and strain rate plays a crucial role in the budget of the Reynolds stress tensor $\overline{u_i u_j}$. Since, in an incompressible flow, its trace is zero, it serves to redistribute energy among the normal stresses and to diminish the correlation between off-diagonal components. There are two

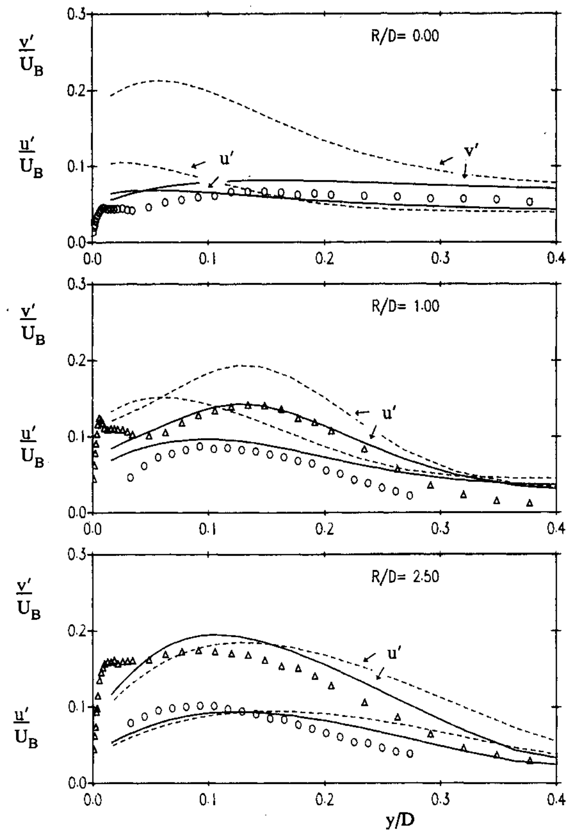

Fig. 1 Profiles of rms velocity fluctuations normal to plate (v') and radially (u') normalized by bulk velocity in pipe U_B: Δ, ○ u', v' experiment[11]; lines, computations: basic model, Eq. (4), --------; and new wall reflection model, Eq. (5), ———.

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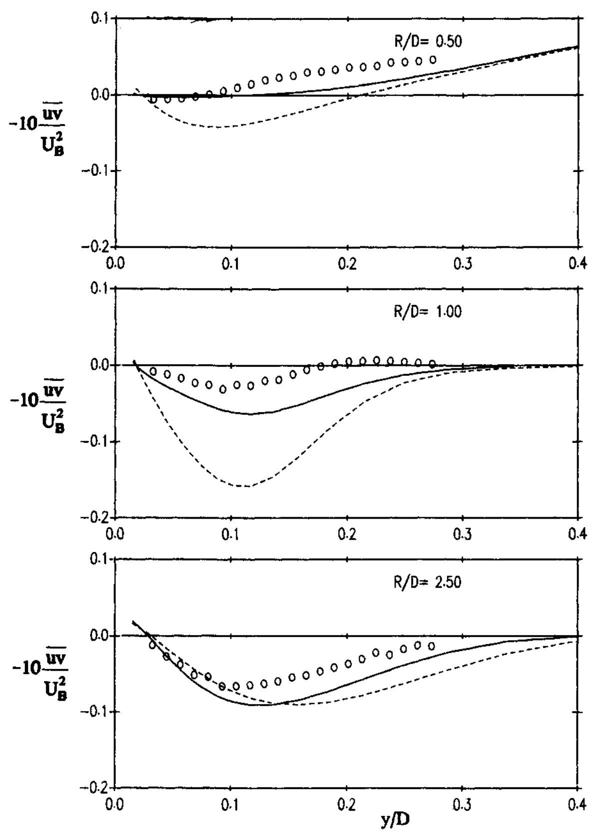

Fig. 2 Profiles of normalized shear stress ($\overline{uv}/U_B^2$): o experiment[11]; lines, computations; key as Fig. 1.

contributors to this process, one associated with a nonlinear interaction, φ_{ij1} (the "turbulent" part of φ_{ij}), and a second involving mean strains, φ_{ij2} (the "mean-strain" or "rapid" part). Although other more elaborate and better founded representations of these processes have been proposed recently, at a practical level most computations of inhomogeneous flows adopt the following admirably simple forms due to Rotta[7] and Naot et al.[8]:

$$\varphi_{ij1} = -c_1 \frac{\epsilon}{k}\left(\overline{u_iu_j} - \frac{1}{3}\delta_{ij}\overline{u_ku_k}\right) \tag{1}$$

$$\varphi_{ij2} = -c_2\left(P_{ij} - \frac{1}{3}\delta_{ij}P_{kk}\right) \tag{2}$$

where k is the turbulent kinetic energy, ϵ its rate of viscous dissipation, and in a nonaccelerating reference frame, P_{ij} denotes the shear production of Reynolds stress

$$P_{ij} \equiv -\left(\overline{u_iu_k}\frac{\partial U_j}{\partial x_k} + \overline{u_ju_k}\frac{\partial U_i}{\partial x_k}\right)$$

The values recommended by Ref. 2 for the coefficients in Eqs. (1) and (2) are $c_1 = 1.8$ and $c_2 = 0.6$.

The presence of a rigid boundary necessitates a correction to the previous equations to reduce the intensity of velocity fluctuations normal to the wall in the vicinity of the surface. Reference 2 adopts the following form guided by an earlier proposal of Shir[9]:

$$\varphi_{ij1}^w = 0.5\frac{\epsilon}{k}\left(\overline{u_\ell u_m}\, n_\ell n_m \delta_{ij} - \frac{3}{2}\overline{u_iu_\ell}\, n_j n_\ell - \frac{3}{2}\overline{u_ju_\ell}\, n_i n_\ell\right)\left(\frac{\ell}{2.5y}\right) \tag{3}$$

$$\varphi_{ij2}^w = 0.3\left(\varphi_{\ell m2}\, n_\ell n_m \delta_{ij} - \frac{3}{2}\varphi_{i\ell 2}\, n_j n_\ell - \frac{3}{2}\varphi_{j\ell 2}\, n_i n_\ell\right)\left(\frac{\ell}{2.5y}\right) \tag{4}$$

where the various n_k are unit vectors normal to the wall and ℓ is the length scale $k^{3/2}/\epsilon$. The form of Eq. (3) always ensures that the correction to φ_{ij1} acts as a sink in the budget of the component normal to the wall and as a source in the other two components. The action of φ_{ij2}^w, however, depends on the nature of the mean-strain field. In a simple shear, φ_{ij2} acts to diminish the effective generation of $\overline{u^2}$ by redistributing it equally to the normal stresses in the x_2 and x_3 directions. The effect of φ_{ij2}^w is to oppose this redistribution and hence to reduce the energy flow rate to $\overline{v^2}$. It thus acts in the same sense as φ_{ij1}^w. In the case of a stagnation flow, the generation term acts to increase $\overline{v^2}$, the component normal to the wall. This effect is tempered by φ_{ij2} as it redistributes the generation. The action of φ_{ij2}^w, however, is to reduce the net φ_{ij2} exchange between normal stress components and hence to increase $\overline{v^2}$—the opposite effect from that desired. Although the formulation of Ref. 2 is spectacularly bad in a stagnation flow, other proposals for the mean-strain part of the wall-reflection process are also unsatisfactory in such strain fields.

In seeking a remedy to the erroneous behavior, we have considered all possible terms involving linear products of the velocity gradient and Reynolds stress tensors (details are omitted here but are given in Ref. 3). Care has been taken to ensure the desired action of φ_{ij2}^w in both the case of shear and stagnating flows. The simplest formulation we have found meeting these requirements is

$$\varphi_{ij2}^w = -0.08\frac{\partial U_\ell}{\partial x_m}\overline{u_\ell u_m}\,(\delta_{ij} - 3n_in_j)\ell/(2.5y) - 0.1ka_{\ell m}\left(\frac{\partial U_k}{\partial x_m}n_\ell n_k\delta_{ij} - \frac{3}{2}\frac{\partial U_i}{\partial x_m}n_\ell n_j - \frac{3}{2}\frac{\partial U_j}{\partial x_m}n_\ell n_i\right)\ell/(2.5y) + 0.4\,k\frac{\partial U_\ell}{\partial x_m}n_\ell n_m\left(n_in_j - \frac{1}{3}\delta_{ij}\right)\ell/(2.5y) \tag{5}$$

where

$$a_{ij} \equiv (\overline{u_iu_j} - \frac{1}{3}\delta_{ij}\overline{u_ku_k})/k$$

In a simple shear, only the first of the three terms in Eq. (5) affects the normal stresses: a fraction of the turbulent kinetic energy generation is "redistributed" from the component normal to the wall to the other components. The final term is nonzero only in the case of normal straining, which is the most powerful element in the impinging jet.

Test Case and Computed Behavior

The performance of Eq. (5) has been assessed by applying an adapted version of the elliptic solver TEAM[10] to the axisymmetric impinging jet with the jet discharge two diameters above the impingement surface, the flow at exit from the jet delivery pipe being fully developed at a Reynolds number of 23×10^3. The experimental data are those of Cooper et al.[11] In addition to the mean flow equations, transport equations are solved for the four nonzero components of Reynolds stress and for the energy dissipation rate ϵ. The closure is, with a single exception, identical to the widely used basic model (see,

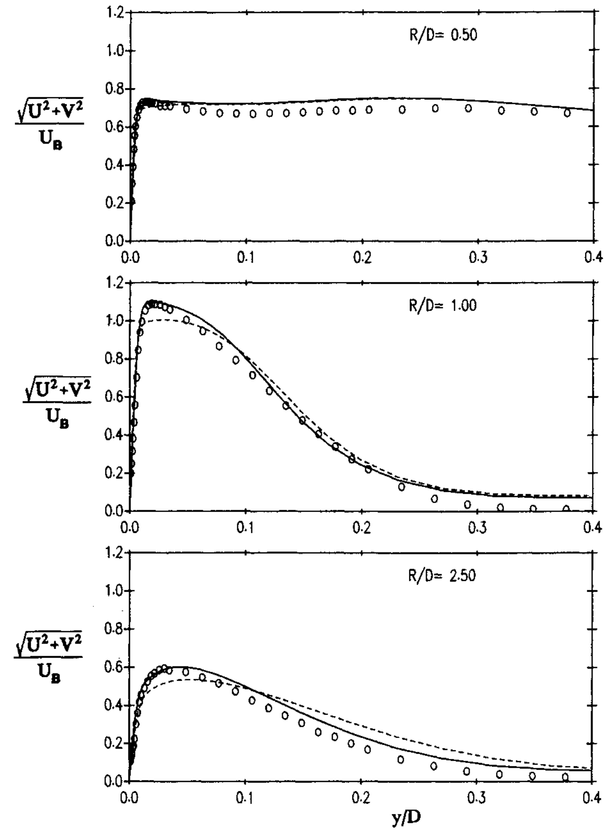

Fig. 3 Profiles of normalized mean velocity; key as Fig. 1.

for example Refs. 12 and 13). The exception is that the new model for φ_{ij2}^{w}, Eq. (5), is adopted in place of Eq. (4).

Across the viscous sublayer region, the previous closure is not valid because of, among other things, the importance of viscous effects. Instead, a low Reynolds number k-ϵ model is employed.[14] The turbulence model in this sublayer exerts little effect beyond the velocity maximum; thus the behavior predicted in the fully turbulent region is a consequence of the second-moment closure adopted.

The computational domain that extends to 0.5 diameters above the jet discharge and to a radial distance of six jet diameters is covered by a nonuniform 70 (radial) × 80 (axial) node mesh. With the third-order QUICK scheme,[15] used here for discretizing momentum transport, purely numerical errors are negligible with this node density.[3] The computations have been performed on an Alliant FX2800 computer configured to allow the parallel use of 10 processors. The CPU time was approximately 0.75 s per iteration, with around 1000 iterations being required for a converged solution.

Figures 1-3 compare the behavior predicted with the new and the standard wall-reflection models with the experimental data. Figure 1 shows the rms fluctuating velocities normal and parallel to the plate at three radial positions. On the symmetry axis ($R/D = 0.0$), Eq. (4) gives fluctuating velocities normal to the wall twice as large as those parallel to it. The new model, however, greatly reduces this difference and, indeed, reverses it very close to the wall. By $R/D = 1.0$, the streamwise component of fluctuating velocity has established itself as the larger, and the differences between the two sets of predictions have begun to diminish. Finally, at $R/D = 2.5$, both schemes return broadly the same peak levels of fluctuating velocity in close agreement with the measurements except for $\overline{u^2}$ in the vicinity of the wall. In this region, it seems quite possible that the matching of the outer-layer second-moment closure with the eddy viscosity model across the sublayer was having a deleterious effect on the normal stress profiles.

The shear stress profiles in Fig. 2 further confirm the superior predicted behavior when the new wall correction is employed. The discrepancies with Eq. (4) are particularly noticeable at $R/D = 0.5$: a shear stress of the wrong sign is predicted over a significant part of the profile. Finally, Fig. 3 shows the resultant effects on the mean velocity profile. The two sets of predicted velocity profiles are almost identical near the stagnation point despite the large differences in the Reynolds stresses because the flow in this region is predominantly determined by the highly nonuniform pressure field. Farther from the stagnation point, however, the peak velocity in the wall jet decays too rapidly with Eq. (4), whereas the present proposal captures the development with reasonable fidelity.

Conclusion

It is recommended that, when computing impinging-type flows, in place of the usual model of φ_{ij2}^{w} [Eq. (4)] the present proposal—Eq. (5)—should be adopted instead.

Acknowledgments

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